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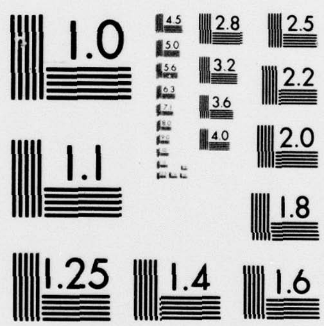

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Continuation Methods for Stability Analysis of Multivariable Feedback Systems*

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Abstract

Techniques for implementation of a Nyquist stability result for a linear time invariant multivariable feedback system are described. The approach is based on continuation methods for computing the system's eigenvalue loci.

I. INTRODUCTION

The classical Nyquist stability criterion for single-input single-output, linear time-invariant feedback systems has only recently been generalized to multivariable feedback systems [1,2]. Stability theorems are expressed in terms of the eigenvalue loci of the open loop transfer function $G(s)$ of the system. In particular if $G(s)$ is stable, i.e., $G(s)$ has no poles in the right half of the s-plane or on the $j\omega$-axis, then a linear time-invariant multivariable feedback system with n inputs and n outputs is stable if and only if its generalized Nyquist plots (union of eigenvalue loci) does not pass through or encircle the (-1, 0) point [1].

In order to apply the multivariable Nyquist criterion, it is thus necessary to compute the eigenvalue loci as a function of frequency. For a given frequency, the eigenvalues can be calculated by using classical techniques. Since the eigenvalues are functions of frequency, normally one would have to repeat the entire computational procedure for each frequency. In the actual stability analysis, this repetition is however, impractical. Our approach to the stability analysis of multivariable feedback systems is based on continuation methods. The basic idea of all continuation methods is to convert the solution of a parameterized family of algebraic problems into the solution of a differential equation. Then if one can find the solution of an initial problem by using classical methods the solutions to the other problems can be obtained by integrating the associated differential equation with the initial solution as an initial condition.

II. EIGENVECTOR APPROACH

Our first method is based on the approach described by Faddeev and Fadeeva [3] and Van Ness et. al. [4]. A differential equation is written with the eigenvalues as dependent variables and the frequency as variable parameter. We then compute a set of initial eigenvalues by classical analysis techniques and integrate the resulted differential equation to obtain the required eigenvalues for each frequency. The eigenvalues $\lambda_i(\omega)$ of $G(j\omega)$ and their complex conjugates $\bar{\lambda}_i(\omega)$ satisfy

$$G(j\omega)X_i(\omega) = \lambda_i(\omega)X_i(\omega) \quad i=1,2,\ldots,n \qquad (1)$$

and

$$G^*(j\omega)V_i(\omega) = \bar{\lambda}_i(\omega)V_i(\omega) \quad i=1,2,\ldots,n \qquad (2)$$

where $X_i(\omega)$ and $V_i(\omega)$ are the corresponding eigenvectors of $\lambda_i(\omega)$ and $\bar{\lambda}_i(\omega)$ respectively, and $G^*(j\omega)$ is the complex conjugate transpose matrix of $G(j\omega)$.

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We differentiate (1) with respect to ω to yield

$$\frac{d\lambda_i}{d\omega} = \frac{\langle \frac{dG}{d\omega} X_i, V_i \rangle}{\langle X_i, V_i \rangle}, \quad i = 1, 2, \ldots, n. \tag{3}$$

The differential equations involving X_i and V_i are obtained as

$$\frac{dX_i}{d\omega} = \sum_{j=1}^{n} \alpha_{ij} X_j, \quad i = 1, 2, \ldots, n. \tag{4}$$

$$\frac{dV_i}{d\omega} = \sum_{j=1}^{n} \beta_{ij} V_j, \quad i = 1, 2, \ldots, n. \tag{5}$$

where

$$\alpha_{ii} = 0, \quad \alpha_{ij} = \frac{\langle \frac{dG}{d\omega} X_i, V_j \rangle}{(\lambda_i - \lambda_j)\langle X_j, V_j \rangle} \quad i \neq j. \tag{6}$$

$$\beta_{ii} = 0, \quad \beta_{ij} = \frac{\langle \frac{dV_i}{d\omega}, X_j \rangle}{\langle V_j, X_j \rangle} \quad i \neq j. \tag{7}$$

Starting with a set of predetermined initial conditions $\lambda_i(0) = \lambda_{io}$, $X_i(0) = X_{io}$ and $V_i(0) = V_{io}$ for $i = 1, 2, \ldots, n$, we integrate (3), (4) and (5) to obtain the required eignevalues for each frequency. The eigenvalue loci are computed in a continuous manner by numerical integration.

III. JACOBIAN METHOD

For an nth order system, the above algorithm requires the numerical integration of a set of 3n equations and the computation of two sets of unwanted variables--namely the eigenvectors X_i and V_i. These disadvantage, can easily be avoided if the characteristic equation for the multivariable feedback system can be predetermined. A much simpler method can be formulated based on the approach for finding multiple solutions for a nonlinear equation developed by Chao et. al. [5].

Let the characteristic equation of $G(j\omega)$ be given by an nth order polynomial in eigenvalue λ with complex coefficients

$$f[\lambda(\omega)] = |\lambda I - G(j\omega)| = 0. \tag{8}$$

Instead of solving (8) directly for each frequency, we consider two simultaneous differential equations of the form

$$\frac{df}{dt} = -f(t) \quad f(0) = f[\lambda(\omega_0)] = 0$$
$$\frac{d\omega}{dt} = \pm 1 \quad \omega(0) = \omega_0. \tag{9}$$

Assuming the nonsingularity of the Jacobian Matrix

$$J = \begin{bmatrix} \frac{\partial f}{\partial \lambda} & \frac{\partial f}{\partial \omega} \\ \frac{\partial \omega}{\partial \lambda} & \frac{\partial \omega}{\partial \omega} \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial \lambda} & \frac{\partial f}{\partial \omega} \\ 0 & 1 \end{bmatrix}, \tag{10}$$

in the x-ω space the algorithm (9) reduces to

$$\begin{bmatrix} \frac{d\lambda}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = J^{-1} \begin{bmatrix} -f \\ \pm 1 \end{bmatrix}; \quad \begin{bmatrix} f(0) \\ \omega(0) \end{bmatrix} = \begin{bmatrix} 0 \\ \omega_0 \end{bmatrix}. \tag{11}$$

It is seen from the solution of (9)

$$f(t) = 0e^{-t} \equiv 0 \qquad \omega = \pm t. \tag{12}$$

that for any admissible pair of ω_0 and $\lambda(\omega_0)$ satisfying (8), the corresponding trajectory will remain on the solution curve f=0 as ω changes. The + or - sign is chosen depending on whether one would like to increase or decrease ω. Equation (11) may now be solved by any numerical integration techniques and the eigenvalue loci can be traced automatically by integrating only a second order differential system.

IV. EXAMPLE

To illustrate the approaches presented, consider a linear time-invariant, multivariable feedback system with open loop transfer function characterized by

$$G(s) = \begin{bmatrix} 4 & \frac{k}{s+2} \\ \frac{s+2}{s+1} & 4 \end{bmatrix} \tag{13}$$

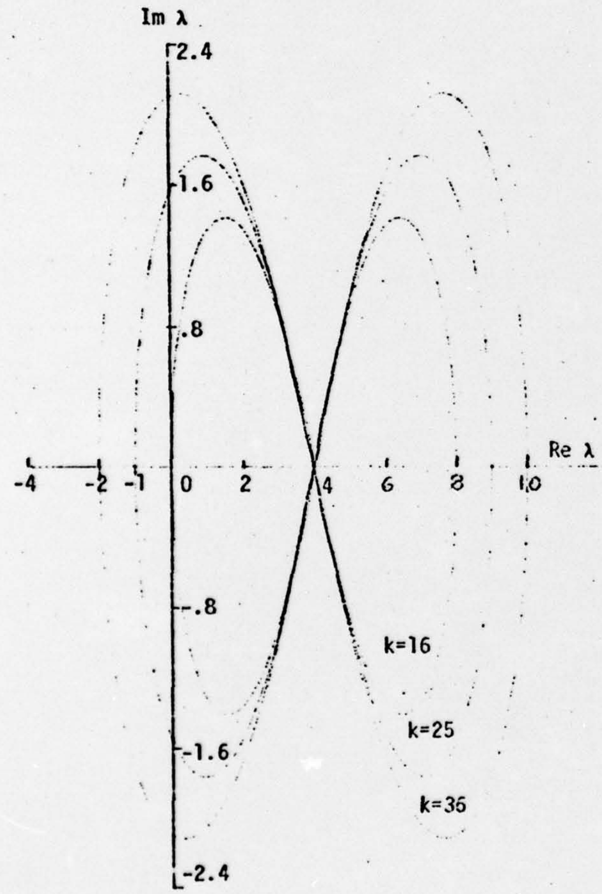
for which the characteristic equation is given by

$$f[\lambda(\omega)] = \lambda^2 - 8\lambda + \frac{16 - k + 16\omega^2}{1 + \omega^2} + j\frac{k\omega}{1 + \omega^2} = 0. \tag{14}$$

The generalized Nyquist plots shown in the accompanied figure for the cases where k=16 and 36 are obtained by applying the eigenvector approach where

as in the critical case, k=25, the Jacobian method has been used.

In all three cases, the equations are integrated using Euler's method with a step size of 0.01. It is seen from the figure that the system is stable for k<25 since the generalized Nyquist plots do not encircle -1 point.



FIGURE

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